Prove the derivative of $\tanh^{-1} x$ without using the logarithmic definition of $\tanh^{-1} x$. SCORE: ______ / 4 PTS

You may use the derivatives of the non-inverse hyperbolic functions that were listed in your textbook without proving them.

NOTE: You must prove the Pythagorean–like identity for $\tanh x$ if you wish to use it.

$y = \tanh^{-1} x$

$x = \tanh y$ $\frac{1}{2}$

$1 = \operatorname{sech}^2 y \frac{dy}{dx}$ 1

$\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y}$ $\frac{1}{2}$

$= \frac{1}{1 - \tanh^2 y} = \frac{1}{1 - x^2}$ $\frac{1}{2}$

$\cosh^2 y - \sinh^2 y = 1$

$\frac{\cosh^2 y - \sinh^2 y}{\cosh^2 y} = \frac{1}{\cosh^2 y}$ 1

$1 - \tanh^2 y = \operatorname{sech}^2 y$ $\frac{1}{2}$

Find $\dfrac{d}{dx}\tanh^{-1}(\operatorname{sech} x)$.

SCORE: ______ / 3 PTS

You may use the hyperbolic identities and derivatives from your textbook without proving them.

$$1 \quad \boxed{\frac{1}{1-\operatorname{sech}^2 x}} \cdot \underline{-\operatorname{sech} x \tanh x} \quad 1$$

$$= \frac{1}{\tanh^2 x} \cdot -\operatorname{sech} x \cancel{\tanh} x$$

$$= \boxed{\frac{-\operatorname{sech} x}{\tanh x}} \ \tfrac{1}{2} = \frac{-\operatorname{sech} x}{\tanh x} \cdot \frac{\cosh x}{\cosh x} = -\frac{1}{\sinh x} = \underline{-\operatorname{csch} x} \ \tfrac{1}{2}$$

Find $\lim_{x \to 0^-} \operatorname{csch} x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.** SCORE: ______ / 2 PTS

$$\lim_{x \to 0^-} \frac{2}{e^x - e^{-x}} = -\infty \quad 1$$

AS $x \to 0^-$, $e^x \to 1^-$ AND $e^{-x} \to 1^+$, SO $e^x - e^{-x} \to 0^-$ 1

$$\frac{2}{0^-} \to -\infty$$

Write an algebraic expression to approximate the area under $f(x) = \ln x$ over the interval $[2, 7]$ using a left hand sum and n subintervals (as shown in class). **Do NOT evaluate the expression. Your answer must not use $f(\,)$ notation.** **SCORE: ______ / 3 PTS**

$$\sum_{i=0}^{n-1} f(a+i\Delta x)\Delta x$$

$a=2$

$\Delta x = \frac{7-2}{n} = \frac{5}{n}$

$$= \sum_{i=0}^{n-1} f\left(2+\frac{5i}{n}\right)\frac{5}{n}$$

1 1 1

$$= \boxed{\sum_{i=0}^{n-1}}\ \boxed{\frac{5}{n}}\ \boxed{\ln\left(2+\frac{5i}{n}\right)}$$

OR

$$\ln(2)\cdot\frac{5}{n} + \ln\left(2+\frac{5}{n}\right)\cdot\frac{5}{n} + \ln\left(2+\frac{10}{n}\right)\cdot\frac{5}{n} + \ldots + \ln\left(2+\frac{5(n-1)}{n}\right)\cdot\frac{5}{n}$$

If $\coth x = -3$, find $\operatorname{sech} x$ and $\sinh x$. **SCORE: ______ / 5 PTS**

$$\tanh x = \frac{1}{\coth x} = \boxed{-\frac{1}{3}} \quad 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\boxed{1 - \left(-\frac{1}{3}\right)^2 = \operatorname{sech}^2 x} \quad 1$$

$$\operatorname{sech}^2 x = \frac{8}{9}$$

$$\boxed{\operatorname{sech} x = \frac{2\sqrt{2}}{3}} \quad 1$$

SINCE $\operatorname{sech} x > 0$

FOR ALL x

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{3}{2\sqrt{2}}$$

$$1 \quad = \boxed{\frac{3\sqrt{2}}{4}}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

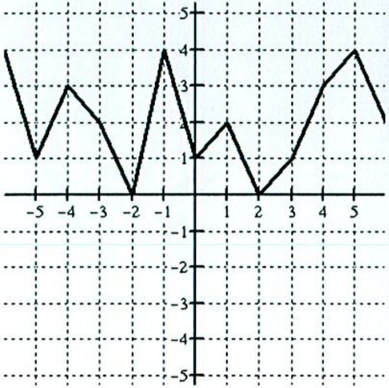
$$\sinh x = \tanh x \cosh x$$

$$= -\frac{1}{3} \cdot \frac{3\sqrt{2}}{4}$$

$$= \boxed{-\frac{\sqrt{2}}{4}} \quad 1$$

Estimate the area under the function shown on the right over the interval $[-6, 2]$ using the left hand sum with 4 equal width subintervals.

SCORE: ______ / 3 PTS



$\Delta x = \frac{2 - -6}{4} = \underbrace{2}$ $\frac{1}{2}$

INTERVALS : $[-6, -4]\ [-4, -2]\ [-2, 0]\ [0, 2]$

$\text{AREA} \approx f(-6) \cdot 2 + f(-4) \cdot 2 + f(-2) \cdot 2 + f(0) \cdot 2$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$= \underbrace{4 \cdot 2} + \underbrace{3 \cdot 2} + \underbrace{0 \cdot 2} + \underbrace{1 \cdot 2}$

$= \underbrace{16}$ $\frac{1}{2}$